PERGAMON

International Journal of Heat and Mass Transfer 44 (2001) 169–180

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Extended performance evaluation criteria for enhanced heat transfer surfaces: heat transfer through ducts with constant heat flux

Ventsislav Zimparov

Gabrovo Technical University, 4, H. Dimitar St., BG-5300 Gabrovo, Bulgaria

Received 7 February 2000

Abstract

Extended performance evaluation criteria equations for enhanced heat transfer surfaces based on the entropy production theorem are developed to include the effect of fluid temperature variation along the length of a tubular heat exchanger. The equations originate from various design constraints and generalize the performance evaluation criteria (PEC) for enhanced heat transfer techniques obtained by means of first law analysis. The general evaluation criteria add new information to Bejan's entropy generation minimization method assessing two objectives simultaneously. The application of this more comprehensive treatment of PEC compared with previous references is illustrated by the analysis of heat transfer and fluid friction characteristics of 10 spirally corrugated tubes assessing the benefit of these tubes as an augmentation technique. The results for different design constraints show that the optimum rib-height-to-diameter ratio (e/D) for these tubes is about 0.04.

Keywords: Performance evaluation criteria; Enhanced heat transfer; Single-phase flow in ducts; Entropy generation minimization; Constant heat flux

1. Introduction

The performance of conventional heat exchangers, for single-phase flows in particular, can be substantially improved by many augmentation techniques resulting in the design systems. Heat transfer enhancement devices are commonly employed to improve the performance of an existing heat exchanger or to reduce the size and cost of a proposed heat exchanger. A performance comparison of effectiveness for various types of enhanced surfaces may lead to selection criteria for designers and users. On the basis of the first law analysis, several authors [1–5] have proposed performance evaluation criteria (PEC) which define the performance benefits of an exchanger having enhanced surfaces, relative to a standard exchanger with smooth surfaces subject to various design constraints. The first systematic studies, developed for single-phase flows, which present several alternative ratios, appear to be those of Webb and Eckert [1] and Bergles et al. [2,3]. Further evolution of the number of PEC was made by Webb [4,5] involving 12 criteria.

On the other hand, it is well established that the minimization of the entropy generation in any process leads to the conservation of useful energy. In a

E-mail address: vdzim@tugab.bg (V. Zimparov).

PII: S0017-9310(00)00074-0

Nomenclature

A heat transfer surface area (m^2)
c_p specific heat capacity (J kg^{-1} K^{-1})
D tube diameter (m)
e ridge height (m)
h specific enthalpy (J kg^{-1})
k thermal conductivity (W m^{-1} K^{-1})
L tube length (m)
$\dot{m}$ mass flow rate in tube (kg s^{-1})
N_t number of tubes
P pumping power (W)
p pitch of ridging (m)
Δp pressure drop (Pa)
$\dot{Q}_t$ heat transfer rate from tube (W)
$\dot{S}_{gen}$ rate of entropy generation (W K^{-1})
s specific entropy (J kg^{-1} K^{-1})
T fluid temperature (K)
ΔT wall-to-fluid temperature difference (K)
u_m mean flow velocity (m s^{-1})
U overall heat transfer coefficient (W m^{-2} K^{-1})
v specific volume of fluid (m^3 kg^{-1})
W mass flow rate in heat exchanger (kg s^{-1})
x axial distance along the tube (m)

Greek symbols
α heat transfer coefficient (W m^{-2} K^{-1})
β helix angle of rib (deg)
μ dynamic viscosity (Pa s)
ρ fluid density (kg m^{-3})

Dimensionless groups
A_* dimensionless heat transfer surface, A_R/A_S
D_* dimensionless tube diameter, D_R/D_S
L_* dimensionless tube length, L_R/L_S
f Fanning friction factor, $2\tau_w/(\rho u_m^2)$
Nu Nusselt number, $\alpha_i D/k_f$
N_S augmentation entropy generation number
N_* ratio of number of tubes, $N_{t,R}/N_{t,S}$
Pr Prandtl number, $\mu c_p/k_f$
P_* dimensionless pumping power, P_R/P_S
Q_* dimensionless heat transfer rate, $\dot{Q}_R/\dot{Q}_S$
Re Reynolds number, $\rho u_m D/\mu$
St Stanton number, $\alpha_i/(\rho u_m c_p)$
ΔT_i^* dimensionless inlet temperature difference between hot and cold streams, $\Delta T_{i,R}/\Delta T_{i,S}$
$u_{m,*}$ dimensionless flow velocity, $u_{m,R}/u_{m,S}$
W_* dimensionless mass flow rate, W_R/W_S
β_* $\beta/90$
ε_* ratio of heat exchanger effectivenesses, $\varepsilon_R/\varepsilon_S$
τ dimensionless temperature difference, $\Delta T/T$
ϕ_0 irreversibility distribution ratio

Subscripts
f fluid
i inside
i value at $x = 0$
m mean value
R rough tube
S smooth tube
o outside
o value at $x = L$

heat exchanger unit, entropy is generated by the heat transferred due to temperature difference and by the irreversible dissipation of kinetic energy due to fluid friction. Heat transfer enhancement devices increase the rate of heat transfer, but they also increase the friction factor associated with the flow. This raises the question of how to employ enhancement techniques in order to minimize the overall entropy generation associated with the heat exchanger operation.

A solid thermodynamic basis to evaluate the merit of augmentation techniques by second law analysis has been proposed by Bejan [6,9] in developing the entropy generation minimization (EGM) method also known as "thermodynamic optimization". The ultimate purpose is to evaluate the advantage of a given augmentation technique by comparing the rates of entropy generation in an augmented duct and in a reference smooth one. This method of optimizing was applied to the design of two augmentation techniques: rough surfaces and swirl promoters [6–8].

Further expansion of the number of PEC [1–5] is possible if the entropy generation as a variable is included, as suggested by Nelson and Bergles [10]. On the basis of preliminary results on 86 PEC, few criteria named Super PEC have been identified that characterize most of the other PEC. These Super PEC represent the same design objectives that have been discussed and applied earlier [1–5]: increase in heat transfer while not changing the heat exchanger geometry or mass flow rate; decrease in pumping power with the same geometry and thermal performance (flow rate decreases); decrease in exchanger size for the same flow rate, pumping power and heat transfer.

The analysis of Refs. [7–9] is performed for constraints $W_* = 1$ and $Q_* = 1$ which corresponds to the case FG-1b [4,5]. Other publications on this subject are Refs. [11,12]. One of the problems discussed is how to enhance the heat transfer in order to reduce the temperature difference which is the driving force for the heat transfer process with the constraints of the case FG-1b [4,5] or with the following constraints: fixed basic geometry, heat duty and pressure drop [12]. Another problem discussed in Ref. [11] is to enhance the heat transfer in order to reduce the surface area of the unit with the constraints corresponding to the case FN-1 [4,5].

The method proposed by Webb and Bergles [1–5], however, does not allow the assessment of two or more objectives simultaneously and does not take into account the entropy generation and one-way destruction of exergy. In many of their cases, Nelson and Bergles [10] also assess one objective taking the rate of entropy production as a constraint. The method of Bejan [6–9] does not include the effect of variation in fluid temperature similar to that present in tubular heat exchangers.

Nad and Kumar [13] modified Bejan's entropy generation criterion by including the fluid temperature variation along heat transfer passage. Prasad and Shen [14,15] propose an evaluation method based on exergy analysis. The analysis includes the effect of fluid temperature variation along the length of a tubular heat exchanger. The thermodynamic optimum is obtained by minimizing the exergy destruction presented by exergy destruction number (N_E). In addition, other criterion such as heat transfer improvement number (N_H) is introduced. These numbers permit a comparison of the effect of improved heat transfer with increased irreversibility. PEC equations based on the first law analysis [1–5] and entropy production theorem have been developed by Zimparov and Vulchanov [16].

The purpose of this paper is to extend the PEC equations discussed previously [16] including the effect of fluid temperature variation along the length of a tubular heat exchanger and to add new information to Bejan's EGM method, assessing two objectives simultaneously.

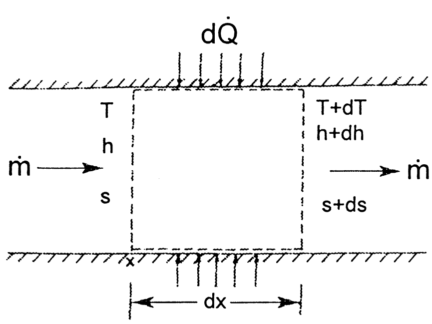

Fig. 1. Control volume for energy and entropy analysis.

2. Equations based on the entropy production theorem

Consider the energy balance of the control volume of length $\mathrm{d}x$ of the duct with constant heat flux (Fig. 1),

$$\mathrm{d}\dot{Q} = \alpha\pi D\,\mathrm{d}x\Delta T = \pi\frac{D^2}{4}\rho u_\mathrm{m} c_\mathrm{p}\,\mathrm{d}T_\mathrm{m}, \tag{1}$$

where $\mathrm{d}\dot{Q}/\mathrm{d}x = \text{constant}$, $\Delta T = \text{constant}$, $\mathrm{d}T_\mathrm{m}/\mathrm{d}x = \text{constant}$ and the fluid and wall temperatures vary linearly with the duct length. Integrating Eq. (1) yields

$$T(x) = T_\mathrm{i} + 4St\frac{\Delta T}{D}x. \tag{2}$$

The temperature of the wall at any instant is $T_\mathrm{w}(x) = T(x) + \Delta T$. Considering an entropy balance in the same control volume, the rate of entropy generation is

$$\mathrm{d}\dot{S}_\mathrm{gen} = \dot{m}\,\mathrm{d}s - \frac{\mathrm{d}\dot{Q}}{T+\Delta T}. \tag{3}$$

Assuming the fluid to be an ideal gas or to be incompressible, $\mathrm{d}h = c_\mathrm{p}\,\mathrm{d}T$, and using the thermodynamic relation $T\,\mathrm{d}s = \mathrm{d}h - v\,\mathrm{d}p$ and $\mathrm{d}\dot{Q} = \dot{m}\,\mathrm{d}h$, Eq. (3) can be written as

$$\begin{aligned}\mathrm{d}\dot{S}_\mathrm{gen} &= \dot{m}\left(c_\mathrm{p}\frac{\mathrm{d}T}{T} - \frac{v}{T}\,\mathrm{d}p\right) - \dot{m}c_\mathrm{p}\frac{\mathrm{d}T}{T+dT} \\ &= \dot{m}c_\mathrm{p}\,\mathrm{d}T\left(\frac{1}{T} - \frac{1}{T+\Delta T}\right) - \dot{m}\frac{v}{T}\,\mathrm{d}p,\end{aligned}$$

or

$$\begin{aligned}\frac{\mathrm{d}\dot{S}_\mathrm{gen}}{\mathrm{d}x} &= \dot{m}c_\mathrm{p}\frac{\mathrm{d}T}{\mathrm{d}x}\frac{\Delta T}{T(T+\Delta T)} + \frac{\dot{m}}{\rho T}\left(-\frac{\mathrm{d}p}{\mathrm{d}x}\right) \\ &= \dot{m}c_\mathrm{p}\frac{\mathrm{d}T}{\mathrm{d}x}\frac{\Delta T}{T^2(1+\tau)} + \frac{\dot{m}}{\rho T}\left(-\frac{\mathrm{d}p}{\mathrm{d}x}\right).\end{aligned} \tag{4}$$

The first and the second terms on the right-hand side of Eq. (4) represent the entropy generation due to heat transfer across finite temperature difference and due to friction, respectively. Substituting Eq. (2) into Eq. (4) and assuming that $\tau \ll 1$, the following equation is obtained

$$\begin{aligned}\frac{\mathrm{d}\dot{S}_\mathrm{gen}}{\mathrm{d}x} &= \dot{m}c_\mathrm{p}4St\frac{\Delta T^2}{D}\frac{1}{\left[T_\mathrm{i} + 4St(\Delta T/D)x\right]^2} \\ &\quad + \frac{\dot{m}}{\rho\left[T_\mathrm{i} + 4St(\Delta T/D)x\right]}\left(-\frac{\mathrm{d}p}{\mathrm{d}x}\right).\end{aligned} \tag{5}$$

Integrating along the length of the duct

$$\dot{S}_{\text{gen}} = \dot{m}c_{\text{p}}\Delta T \frac{4St(\Delta T/D)L}{T_{\text{i}}^2\left[1 + 4St(\Delta T/T_{\text{i}})(L/D)\right]} + \frac{\dot{m}u_{\text{m}}^2 f}{2St\Delta T}\ln\left(1 + 4St\frac{\Delta T}{T_{\text{i}}}\frac{L}{D}\right),$$

or

$$\dot{S}_{\text{gen}} = \frac{\dot{Q}_t\Delta T}{T_{\text{i}}^2}\frac{1}{\left[1 + (T_{\text{o}} - T_{\text{i}})/T_{\text{i}}\right]} + \frac{2\dot{m}fu_{\text{m}}^2 L}{T_{\text{i}}D}\frac{\ln\left[1 + (T_{\text{o}} - T_{\text{i}})/T_{\text{i}}\right]}{(T_{\text{o}} - T_{\text{i}})/T_{\text{i}}}$$
$$= \frac{\dot{Q}_t\Delta T}{T_{\text{i}}^2}\frac{1}{1 + (\Delta T_{\text{m}}/T_{\text{i}})} + \frac{32\dot{m}^3 fL}{\rho^2\pi^2 T_{\text{i}}D^5}\frac{\ln\left[1 + (\Delta T_{\text{m}}/T_{\text{i}})\right]}{\Delta T_{\text{m}}/T_{\text{i}}}, \tag{6}$$

where $-\mathrm{d}p/\mathrm{d}x = 2f\rho u_{\text{m}}^2/D$; $u_{\text{m}} = 4\dot{m}/(\rho\pi D^2)$; $\dot{Q}_{\text{t}} = \alpha\pi DL\Delta T = \dot{m}c_{\text{p}}\Delta T_{\text{m}}$; $St = \alpha/(\rho c_{\text{p}}u_{\text{m}})$ and $\Delta T = [\dot{Q}_{\text{t}}/(4\dot{m}c_{\text{p}}St)]\,D/L$. For tubular full-size heat exchanger, $\dot{Q} = N_{\text{t}}\dot{Q}_{\text{t}}$; $A = \pi DLN_{\text{t}}$; $A_{\text{f}} = [(\pi D^2)/4]N_{\text{t}}$; $W = N_{\text{t}}\dot{m}$ and Eq. (6) yields

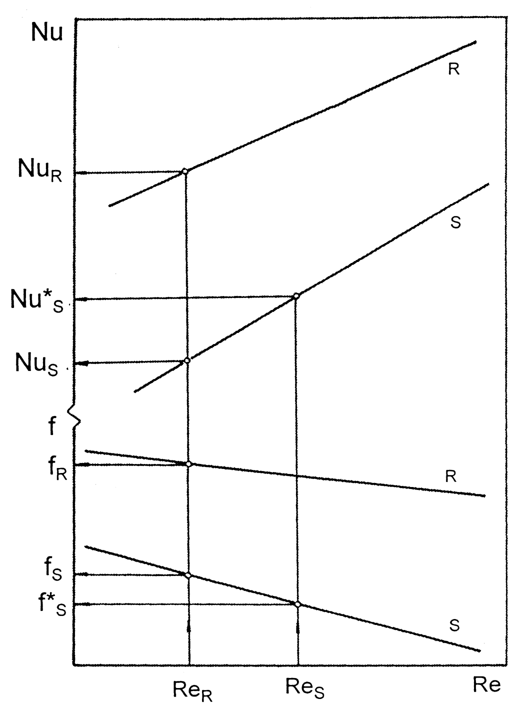

Fig. 2. The behavior of the Stanton number and friction factor vs. Reynolds number.

$$\dot{S}_{\text{gen}} = \frac{\dot{Q}^2}{N_{\text{t}}^2 T_{\text{i}}^2\pi k_{\text{f}}NuL}\frac{1}{\left[1 + (\Delta T_{\text{m}}/T_{\text{i}})\right]} + \frac{32W^3 fL}{N_{\text{t}}^3\rho^2\pi^2 T_{\text{i}}D^5}\frac{\ln\left[1 + (\Delta T_{\text{m}}/T_{\text{i}})\right]}{\Delta T_{\text{m}}/T_{\text{i}}}. \tag{7}$$

Following Bejan [9] the thermodynamic impact of the augmentation technique is defined by the augmentation entropy generation number

$$N_S = \dot{S}_{\text{gen, R}}/\dot{S}_{\text{gen, S}}. \tag{8}$$

Augmentation techniques with $N_S < 1$ are thermodynamically advantageous since in addition to enhancing heat transfer, they reduce the degree of irreversibility of the apparatus. Substituting Eq. (7) into Eq. (8), N_S can be rewritten as

$$N_S = \frac{N_T + \phi_0 N_P}{1 + \phi_0}, \tag{9}$$

where

$$N_T = \frac{\left(\dot{S}_{\text{gen},\,\Delta T}\right)_{\text{R}}}{\left(\dot{S}_{\text{gen},\,\Delta T}\right)_{\text{S}}} = \frac{Q_*^2 Nu_{\text{S}}}{N_*^2 Nu_{\text{R}}L_*}\frac{T_{\text{o, S}}}{T_{\text{o, R}}}, \tag{10}$$

$$\frac{T_{\text{o, S}}}{T_{\text{o, R}}} = \left[\frac{T_{\text{i, S}}}{T_{\text{o, S}}} + \frac{Q_*}{W_*}\left(1 - \frac{T_{\text{i, S}}}{T_{\text{o, S}}}\right)\right]^{-1}, \tag{11}$$

$$N_P = \frac{\left(\dot{S}_{\text{gen},\,\Delta P}\right)_{\text{R}}}{\left(\dot{S}_{\text{gen},\,\Delta P}\right)_{\text{S}}} = \frac{W_*^3 L_*}{N_*^3 D_*^5}f_{\text{R}}/f_{\text{S}} = P_*. \tag{12}$$

In writing Eq. (12) we assumed that $\Delta T_{\text{m}} = T_{\text{o}} - T_{\text{i}}$ is considerably smaller than the inlet absolute temperature T_{i}, namely $\ln[1 + (\Delta T_{\text{m}}/T_{\text{i}})] \cong \Delta T_{\text{m}}/T_{\text{i}} \ll 1$. When the standard heat transfer passage is known, the numerical value of the irreversibility distribution ratio, $\phi_0 = (\dot{S}_{\text{gen},\,\Delta P}/\dot{S}_{\text{gen},\,\Delta T})_{\text{S}}$, describes the thermodynamic mode in which the passage is meant to operate.

$$\phi_0 = \frac{32W_{\text{S}}^3 f_{\text{S}}L_{\text{S}}^2 k_{\text{f}}Nu_{\text{S}}T_{\text{i, S}}}{N_{\text{t, S}}\rho^2\pi D_{\text{S}}^5\dot{Q}_{\text{S}}^2}\left(1 + \frac{\Delta T_{\text{m, S}}}{T_{\text{i, S}}}\right) \tag{13}$$

Keeping in mind that $\dot{Q}_{\text{S}} = \alpha_{\text{S}}\pi D_{\text{S}}L_{\text{S}}N_{\text{t, S}}\Delta T$ and $W_{\text{S}} = \rho u_{\text{m, S}}N_{\text{t, S}}\pi D_{\text{S}}^2/4$, the expression (13) for the irreversibility distribution ratio ϕ_0 can be simplified

$$\phi_0 = \frac{f_S/2}{St_S}\left(\frac{T_i}{\Delta T}\right)_S^2 \left(\frac{u_m^2}{c_p T_i}\right)_S \left(1 + \frac{\Delta T_m}{T_i}\right)_S$$
$$= \frac{f_S/2}{St_S}\left(\frac{T_i}{\Delta T}\right)_S^2 \left(\frac{u_m^2}{c_p T_i}\right)_S \frac{T_{o,\,S}}{T_{i,\,S}} \qquad (14)$$

which yields a straightforward estimate of ϕ_0.

When an enhanced tube is being considered for replacement of a smooth one, there are many possible effects on performance. The design constraints imposed on the exchanger flow rate and velocity cause key differences among the possible PEC relations [4,5]. The increased friction factor due to augmented surfaces may require a reduced velocity to satisfy a fixed pumping power (or pressure drop) constraint. If the exchanger flow rate is held constant, it may be necessary to increase the flow frontal area to satisfy the pumping power constraint. However, if the mass flow rate is reduced, it is possible to maintain a constant flow frontal area at reduced velocity. In many cases the heat exchanger flow rate is specified and a flow rate reduction is not permitted. Despite the fact that a large number of possible PEC can be defined [10], the PEC, as suggested by Webb and Bergles [4,5] characterize nearly all the PEC and they will be considered in what follows. The equations are developed for tubes of different diameters and heat transfer and friction factors based on the presentation format of performance data for enhanced tubes [17]. The relative equations for single-phase flow inside enhanced tubes are:

$$A_* = N_* L_* D_*, \qquad (15)$$

$$P_* = W_* \Delta p_* = f_R/f_S D_* L_* N_* u_{m,\,*}^3 = \frac{W_*^3 L_*}{N_*^2 D_*^5} f_R/f_S, \qquad (16)$$

$$Q_* = W_* \varepsilon_* \Delta T_i^*, \qquad (17)$$

$$W_* = u_{m,\,*} D_*^2 N_* = \frac{Re_R}{Re_S} D_* N_*, \qquad (18)$$

$$\Delta p_* = f_R/f_S \frac{L_*}{D_*} u_{m,\,*}^2 = f_R/f_S \frac{L_* Re_R^2}{D_*^3 Re_S^2}. \qquad (19)$$

2.1. Fixed geometry criteria (FG)

These criteria may be thought of as a retrofit situation, in which there is a one-for-one replacement of smooth tubes with enhanced ones of the same basic geometry, e.g., tube envelope diameter, tube length, and number of tubes for in-tube flow. The FG-1 cases seek increased heat duty or overall conductance UA for constant exchanger flow rate. The pumping power of the enhanced tube exchanger will increase due to the increased fluid friction characteristics of the augmented surface. For these cases, the constraints $\Delta T_i^* = 1$, $W_* = 1$, $N_* = 1$ and $L_* = 1$ require $Re_S = D_* Re_R$ and $P_* > 1$. One of the most common and amply documented heat transfer augmentation techniques is the surface promoters or "in-tube roughness". Wall roughness has a negligible impact on the flow cross section and hydraulic diameter D_h and in many applications it can be assumed $D_* = 1$. Nevertheless, in some cases thinner walled tubes might be used as replacement to diminish the pressure drop and pumping power increase [18]. If the tube-side velocity is reduced, the values of Nu_S and Nu_R are calculated at different Reynolds numbers, Nu_S for Re_S and Nu_R for Re_R. Fig. 2, pertaining to the important area of internal, single-phase forced convection flow, demonstrates the friction factor and heat transfer coefficients. When the objective is increased heat duty $Q_* > 1$, this corresponds to the case FG-1a [4,5], and the augmentation entropy generation number N_S, Eq. (9), becomes

$$N_S = \frac{1}{1+\phi_0}\left(\frac{Nu_S}{Nu_R} Q_*^2 \frac{T_{o,\,S}}{T_{o,\,R}} + \phi_0 \frac{f_R/f_S}{D_*^5}\right). \qquad (20)$$

If the friction factor and Nusselt number characterizing the smooth surface in the turbulent flow are fitted by

$f_S = 0.079 Re^{-0.25}$ and $Nu_S = 0.023 Re^{0.8} Pr^{0.4}$

the ratios Nu_S/Nu_R and f_R/f_S in Eq. (20) can be expressed (Fig. 2) by

$$\frac{Nu_S(Re_S)}{Nu_R(Re_R)} = \frac{Nu_S(Re_R)}{Nu_R(Re_R)} \frac{Nu_S(Re_S)}{Nu_S(Re_R)} = \frac{Nu_S}{Nu_R}\left(\frac{Re_S}{Re_R}\right)^{0.8}$$
$$= f(Re_R),$$

and

$$\frac{f_R(Re_R)}{f_S(Re_S)} = \frac{f_R}{f_S}\left(\frac{Re_R}{Re_S}\right)^{-0.25} = f(Re_R).$$

For the case FG-1a $Re_S = D_* Re_R$ and Eq. (20) can be rewritten in the form

$$N_S = \frac{1}{1+\phi_0}\left\{\frac{Nu_S}{Nu_R} D_*^{0.8} Q_*^2 \left[\frac{T_{i,\,S}}{T_{o,\,S}} + Q_*\left(1 - \frac{T_{i,\,S}}{T_{o,\,S}}\right)\right]^{-1} + \phi_0 \frac{f_R/f_S}{D_*^{4.75}}\right\} = f(Re_R). \qquad (21)$$

If the objective is $U_R A_R > U_S A_S$ for $Q_* = 1$, the driving LMTD can be reduced. This case corresponds to FG-1b [4,5]. The constraints $N_* = 1$, $L_* = 1$, $W_* = 1$ and

$Q_* = 1$ require $Re_S = D_* Re_R$ and $P_* > 1$. The objective is $\Delta T_i^* < 1$. The augmentation entropy generation number N_S, Eq. (9), becomes

$$N_S = \frac{1}{1+\phi_0}\left(\frac{Nu_S}{Nu_R} D_*^{0.8} + \phi_0 \frac{f_R/f_S}{D_*^{4.75}}\right) = f(Re_R). \tag{22}$$

The FG-2 criteria have the same objectives as FG-1, but require that the augmented tube unit operates at the same pumping power as the reference smooth tube unit. The pumping power is maintained constant by reducing the tube-side velocity and thus the exchanger flow rate. The constraints are: $\Delta T_i^* = 1$, $N_* = 1$, $L_* = 1$, and $P_* = 1$ requiring $W_* < 1$ and $Re_R < Re_S$. When the objective is $Q_* > 1$, case FG-2a [4,5], the augmentation entropy generation number N_S yields

$$N_S = \frac{1}{1+\phi_0}\left\{\frac{Nu_S}{Nu_R}(f_R/f_S)^{0.291} D_*^{-0.582} Q_*^2 \left[\frac{T_{i,S}}{T_{o,S}} + Q_*(f_R/f_S)^{0.364} D_*^{-1.727}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_0\right\} = f(Re_R). \tag{23}$$

Recall also that the constraint $P_* = 1$ requires

$$Re_S = Re_R (f_R/f_S)^{0.364} D_*^{-0.727} = f(Re_R) \tag{24}$$

When the objective is $\Delta T_i^* < 1$, with additional constraint $Q_* = 1$, case FG-2b [4,5], the augmentation entropy generation number N_S is

$$N_S = \frac{1}{1+\phi_0}\left\{\frac{Nu_S}{Nu_R}(f_R/f_S)^{0.291} D_*^{-0.582} \left[\frac{T_{i,S}}{T_{o,S}} + (f_R/f_S)^{0.364} D_*^{-1.727}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_0\right\} = f(Re_R). \tag{25}$$

The case FG-2c, where the objective is $\Delta T_i^* < 1$ with the constraints $N_* = 1$, $L_* = 1$, $Q_* = 1$ and $\Delta p_* = 1$ (pressure drop fixed) is an extension of cases FG-2. The consequences are $W_* < 1$, $P_* < 1$, $Re_R < Re_S$ and the case corresponds to the case B [12]. The constraint $\Delta p_* = 1$ imposes

$$Re_S = Re_R (f_R/f_S)^{0.571} D_*^{-1.714} = f(Re_R), \tag{26}$$

and Eq. (9) yields

$$N_S = \frac{1}{1+\phi_0}\left\{\frac{Nu_S}{Nu_R}(f_R/f_S)^{0.457} D_*^{-1.371} \left[\frac{T_{i,S}}{T_{o,S}} + (f_R/f_S)^{0.571} D_*^{-2.714}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_0 (f_R/f_S)^{-0.571} D_*^{2.714}\right\} = f(Re_R). \tag{27}$$

The third criterion, FG-3 [4,5], attempts to reduce the pumping power for constant heat duty. The constraints and the consequences are $N_* = 1$, $L_* = 1$, $\Delta T_i^* = 1$, $Q_* = 1$, $W_* < 1$ and $Re_R < Re_S$. The objective is $P_* < 1$. In this case

$$Re_S = Re_R \left(\frac{f_R/f_S}{P_*}\right)^{0.364} D_*^{-0.727} = f(Re_R), \tag{28}$$

and Eq. (9) becomes

$$N_S = \frac{1}{1+\phi_0}\left\{\frac{Nu_S}{Nu_R}\left(\frac{f_R/f_S}{P_*}\right)^{0.291} D_*^{-0.582} \left[\frac{T_{i,S}}{T_{o,S}} + \left(\frac{f_R/f_S}{P_*}\right)^{0.364} D_*^{-1.727}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_0 P_*\right\} = f(Re_R). \tag{29}$$

2.2. Fixed flow area criteria (FN)

These criteria maintain fixed flow frontal area. For a shell-and-tube heat exchanger having constant outside diameter tubes, this means that the number of tubes and shell diameter are held constant. These criteria seek reduced surface area or reduced pumping power for constant heat duty. The objective of FN-1 case [4,5] is reduced surface area by reduced tube length, $L_* < 1$, for constant pumping power, $P_* = 1$. The additional constraints are $N_* = 1$, $Q_* = 1$, $\Delta T_i^* = 1$ requiring $W_* < 1$ and $Re_R < Re_S$. In this case the constraint $P_* = 1$ imposes

$$Re_S = Re_R \left(\frac{f_R}{f_S} L_*\right)^{0.364} D_*^{-0.727} = f(Re_R), \tag{30}$$

and the augmentation entropy generation number is

$$N_S = \frac{1}{1+\phi_0}\left\{\frac{Nu_S}{Nu_R}(f_R/f_S)^{0.291} D_*^{-0.582} L_*^{-0.709} \left[\frac{T_{i,S}}{T_{o,S}} + \left(\frac{f_R}{f_S} L*\right)^{0.364} D_*^{-1.727}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_0\right\} = f(Re_R). \tag{31}$$

The objective of FN-2 case [4] (FN-3 [5]), is to reduce

Table 1

Tube No.	Reference	D_o (mm)	D_i (mm)	e (mm)	p (mm)	e/D	p/e	β	E_o
(1) 2a		25.40	23.57	0.271	2.54	0.012	9.4	0.935	1.64
(2) 2b	[19]	25.40	23.57	0.393	6.35	0.017	16.1	0.840	1.29
(3) 2c		25.40	23.57	0.751	12.70	0.032	16.9	0.698	1.09
(4) 6		25.30	23.39	0.886	9.75	0.038	11.0	0.906	1.07
(5) 7	[20]	25.30	23.42	0.775	9.40	0.033	12.1	0.919	1.10
(6) 2200		24.61	21.90	0.439	6.35	0.020	14.5	0.941	1.25
(7) 14		25.38	22.00	0.947	13.46	0.043	14.2	0.878	1.04
(8) 18		27.27	25.20	1.022	10.95	0.041	10.7	0.919	1.07
(9) 33	[21]	27.56	25.62	0.447	6.55	0.017	14.7	0.952	1.20
(10) 34		27.62	25.78	0.628	8.48	0.024	13.5	0.938	1.08

pumping power, $P_* < 1$, with constant heat duty, $Q_* = 1$, and flow rate $W_* = 1$. Other constraints are $N_* = 1, \Delta T_i^* = 1$ and consequently $L_* < 1$ and $Re_S = D_* Re_R$. The equation for the augmentation entropy generation number is

$$N_S = \frac{1}{1+\phi_0}\left(\frac{Nu_S}{Nu_R}\frac{f_R/f_S}{P_*}D_*^{-3.95} + \phi_0 P_*\right) = f(Re_R). \tag{32}$$

2.3. Variable geometry criteria (VG)

In many cases a heat exchanger is designed for a required thermal duty with a specified flow rate. Because the tube-side velocity must be reduced to accommodate the higher friction characteristics of the augmented surface, it is necessary to increase the flow area to maintain constant flow rate. This is accomplished using a greater number of tubes in parallel, or by using the same number of larger diameter tubes. All of the VG cases maintain $W_* = 1$ and permit the exchanger flow frontal area to vary in order to meet the pumping power constraint: $N_* > 1$, $L_* < 1$, $Re_R < Re_S$. Case VG-1 yields reduced surface area $A_* < 1$, for $Q_* = 1, P_* = 1$ and $\Delta T_i^* = 1$. The constraint $P_* = 1$ imposes

$$Re_S = Re_R\left(\frac{f_R}{f_S}A_*\right)^{0.364} D_*^{-1.091} = f(Re_R), \tag{33}$$

and the augmentation entropy generation number becomes

$$N_S = \frac{1}{1+\phi_0}\left[\frac{Nu_S}{Nu_R}(f_R/f_S)^{-0.073}A_*^{-1.073}D_*^{2.218} + \phi_0\right] = f(Re_R). \tag{34}$$

The cases VG-2 [4,5] aim at increased thermal performance ($U_R A_R / U_S A_S$ or $Q_* > 1$) for $A_* = 1$ and $P_* = 1$. They are similar to the cases FG-2. When the objective is $Q_* > 1$, case VG-2a [4,5], an additional constraint is $\Delta T_i^* = 1$. The constraint $P_* = 1$ imposes

$$Re_S = Re_R (f_R/f_S)^{0.364} D_*^{-1.091} = f(Re_R), \tag{35}$$

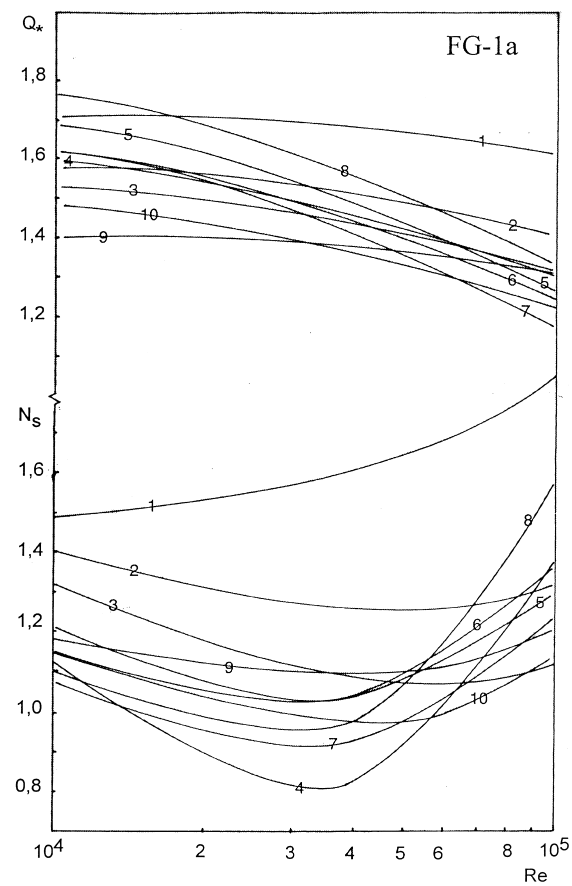

Fig. 3. Increased heat transfer rate and augmentation entropy generation number vs. Reynolds number.

and the augmentation entropy generation number N_S becomes

$$N_S = \frac{1}{1+\phi_0}\left\{\frac{Nu_S}{Nu_R}(f_R/f_S)^{-0.073}D_*^{2.218}Q_*^2\left[\frac{T_{i,S}}{T_{o,S}} + Q_*\left(1-\frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_0\right\} = f(Re_R). \quad (36)$$

When the objective is $\Delta T_i^* < 1$, with additional constraint $Q_* = 1$, case VG-2b [4,5], the augmentation entropy generation number N_S is

$$N_S = \frac{1}{1+\phi_0}\left[\frac{Nu_S}{Nu_R}(f_R/f_S)^{-0.073}D_*^{2.218} + \phi_0\right] = f(Re_R). \quad (37)$$

Case VG-3 [4,5] aims to reduce the pumping power, $P_* < 1$ for $A_* = 1, Q_* = 1$ and $\Delta T_i^* = 1$. It is similar to case FG-3. The Reynolds numbers maintained in the comparable units are defined by

$$Re_S = Re_R\left(\frac{f_R/f_S}{P_*}\right)^{0.364}D_*^{-1.091} = f(Re_R), \quad (38)$$

and the equation for N_S has the form

$$N_S = \frac{1}{1+\phi_0}\left[\frac{Nu_S}{Nu_R}\left(\frac{f_R/f_S}{P_*}\right)^{-0.073}D_*^{2.218} + \phi_0 P_*\right] = f(Re_R). \quad (39)$$

3. Application of PEC equations and discussion

The solution of the performance evaluation criteria

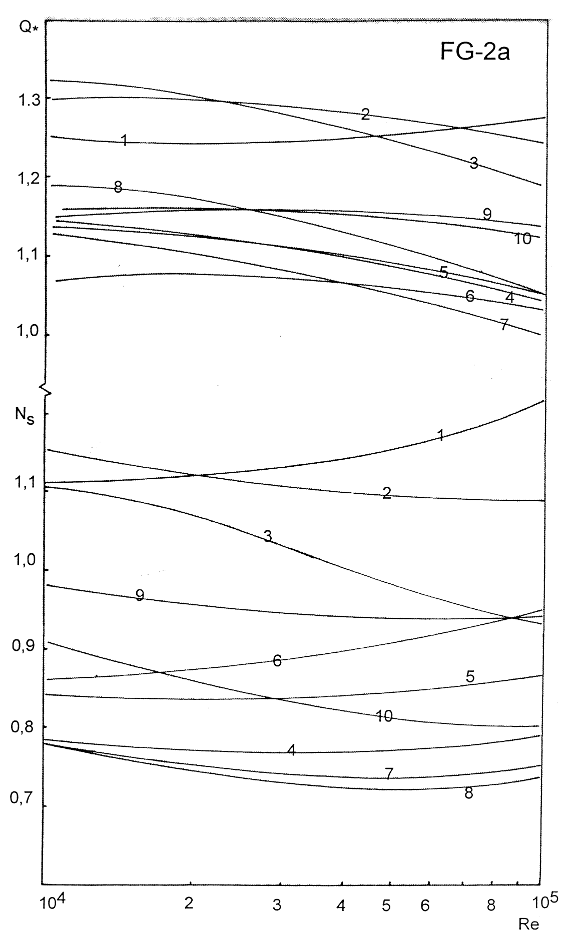

Fig. 4. Increased heat transfer rate and augmentation entropy generation number vs. Reynolds number.

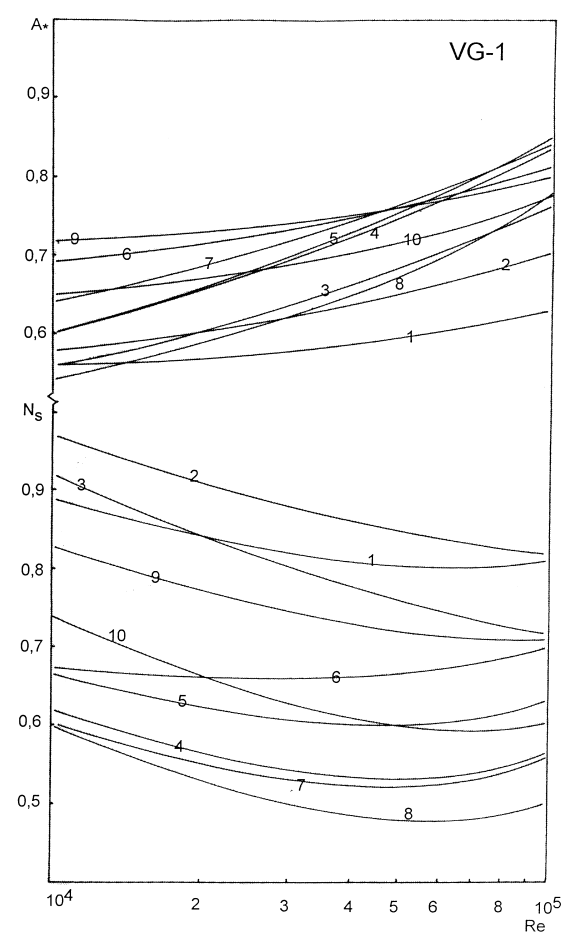

Fig. 5. Reduced heat transfer area and augmentation entropy generation number vs. Reynolds number.

equations described above requires algebraic relations which:

1. Define correlations for $Nu(St)$ and f of the augmented surfaces as a function of Re.
2. Quantify performance objectives and design constraints. This means that the designer should define clearly his or her goal and then solve the equations corresponding to the algebraic relations [4] based on the first law of thermodynamics, to obtain the values of Q_*, A_* or P_* as a function of Re.
3. Calculate the irreversibility distribution ratio ϕ_0 as a function of Re for the reference (smooth) passage.

The performance of heat exchangers, for single-phase flows in particular, can be improved by many augmentation techniques. The most popular and successful technique is augmentation through surface roughness. Transverse or helical repeated ribs are an especially attractive way of creating the surface

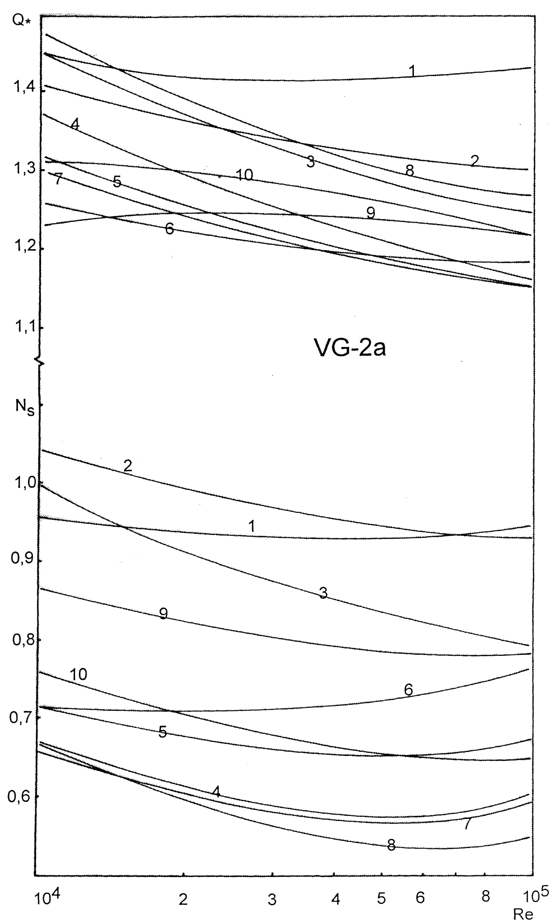

Fig. 6. Increased heat transfer rate and augmentation entropy generation number vs. Reynolds number.

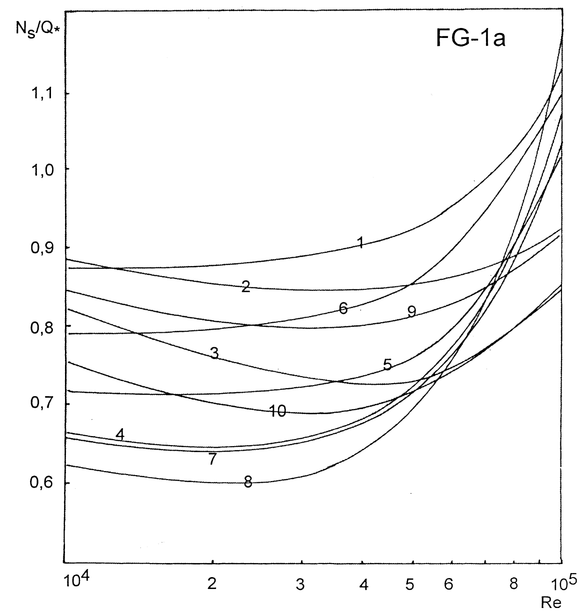

Fig. 7. The ratio N_S/Q_* vs. the Reynolds number.

roughness. The results of this study can be illustrated by the characteristics of the spirally corrugated (roped) tubes for steam condensers obtained through several experimental programs [19–21]. The study of Ravigururajan and Bergles [22,23] based on the first law analysis indicated that the optimum rib-height-to-diameter ratio (e/D) for spirally indented tubes is around 0.02. This conclusion has been verified by using the PEC equations. The geometrical parameters of the tubes considered in this study are presented in Table 1.

The operating conditions of the reference

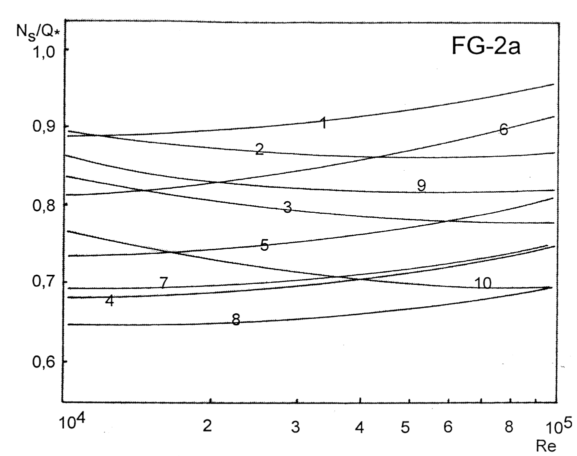

Fig. 8. The ratio N_S/Q_* vs. the Reynolds number.

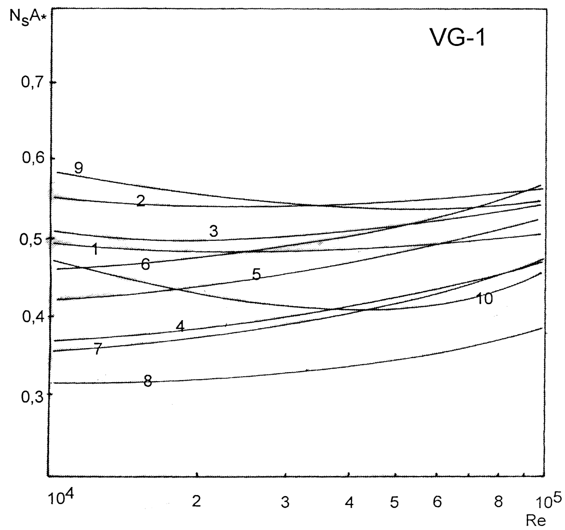

Fig. 9. The group $N_S A_*$ vs. the Reynolds number.

(smooth) passage were chosen as follows $T_{i,S} = 12°C$, $T_{o,S} = 21°C$, $Pr = 7.8$, $10^4 < Re < 10^5$. The corresponding values of ϕ_0 are in the range $6.7 \times 10^{-5} < \phi_0 < 2.4 \times 10^{-3}$ which shows that the channel is dominated by heat transfer irreversibility. The effect of the thermal resistance external to the surface is also took into consideration by including external heat transfer coefficient $\alpha_{o,S} = 10{,}100$ W/m^2 K. The analysis includes the possibility that the augmented exchanger may have an enhanced outer tube surface — $E_o = \alpha_{o,R}/\alpha_{o,S}$. The values of E_o which have been assigned for each tube are presented in Table 1. The fouling resistances on the both sides of the tube wall are neglected.

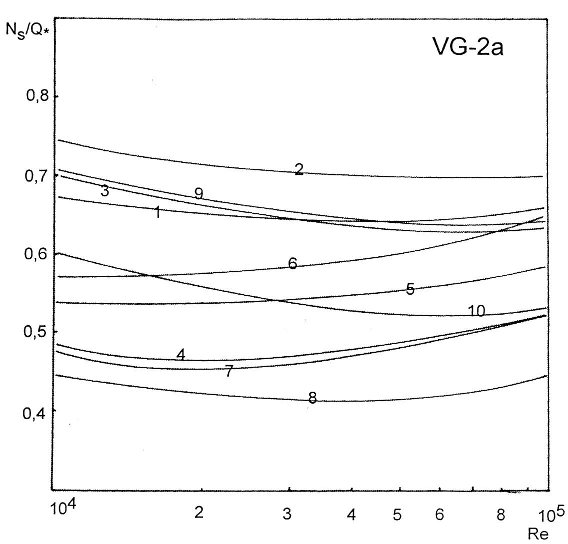

Fig. 10. The ratio N_S/Q_* vs. the Reynolds number.

Fig. 3 represents the case FG-1a [4,5] where the objective is to increase the heat duty, $Q_* > 1$. The values of Q_*, as a function of Re_R are obtained following Webb's treatise on PEC [4]. In this case, the unit with heat transfer enhancing spirally corrugated tubes increases its heat duty, $Q_* > 1$, significantly, e.g. tubes 2a and 2b [19], 18 [21], 6 [20], but most of them do not reduce the destruction of exergy and lead to an increase in the rate of entropy generation. Examining carefully Fig. 3, one may find out that the largest values of N_S have tubes 2a and 2b [19] and the smallest ones have the tubes 6 and 14 [20]. It should also be noted that for all the tubes, the augmentation entropy generation number N_S has a minimum.

In the case FG-2a, the goal is increased heat transfer rate, $Q_* > 1$, with additional constraint for equal pumping power, $P_* = 1$. The variations of Q_* and N_S as a function of Re_R are shown in Fig. 4. As seen, the corrugated tubes 6, 14 [20] and 34, 18 [21] have the smallest values of $N_S < 1$ but do not guarantee the largest heat transfer rate increase. On the other hand, the corrugated tubes 2a, 2b and 2c [19] guarantee the largest values of Q_* but with an increase in the rate of entropy generation, $N_S > 1$.

In the case VG-1, the objective is to reduce surface area $A_* < 1$ with $W_* = 1$ for $Q_* = P_* = 1$. Fig. 5 represents the variations of A_* and N_S with Re_S. In this case, all corrugated tubes reduce the rate of entropy generation, $N_S < 1$. The preferable tubes which have the largest reduction of heat transfer area $A_* < 1$ are 2a, 2b, 2c [19] and 18 [21]. On the other hand, the smallest value of $N_S < 1$ have the tubes 6, 7, 14 [20] and 18 [21].

Fig. 6 illustrates the case VG-2a where the objective is increased heat rate $Q_* > 1$ for $W_* = 1$ and $A_* = P_* = 1$. The best tubes having $Q_* > 1$ are 2a, 2b, 2c [19] and 18 [21], but the smallest values of N_S have the tubes 6, 7, 14 [20] and 18 [21].

The results shown in Figs. 4–6 imply that the evaluation and comparison of the heat transfer augmentation techniques should be made on the basis of both first and second law analysis. Thus, it is possible to determine the thermodynamic optimum in a heat exchanger by minimizing the augmentation entropy generation number compared with the relative increase of heat transfer rate $Q_* > 1$, or relative reduction of heat transfer area $A_* < 1$ or pumping power $P_* < 1$. Consequently, it might be defined ratio N_S/Q_*, and groups $N_S A_*$, $N_S P_* = f(Re_R)$ to connect the two objectives pursued by the first and second law analysis.

Figs. 7–10 show N_S/Q_*–Re_R for the case FG-1a, FG-2a and VG-2a and $N_S A_*$–Re_R for the case VG-1.

For all the cases considered, the best performance have one and the same tubes — 18 [21], 14, 6 [20] and 34 [21] being far superior to others. The benefit of making use of tubes 18 [21] or 14, 6 [20], particularly for $Re < 4.5 \times 10^4$, is significant. Therefore, the analysis using new performance evaluation criteria shows that the optimum rib-height-to-diameter ratio (e/D) for spirally corrugated tubes is about 0.04. Figs. 7–10 also show that for all the tubes considered an optimum Reynolds number, corresponding to minimum entropy generation, can be obtained in each case.

4. Conclusions

The results of the present study can be summarized as follows:

1. Extended PEC equations have been developed to include the effect of fluid temperature variation along the length of a tubular exchanger and to assess heat transfer enhancement techniques based on the entropy production theorem with various constraints imposed. These equations add new PEC for enhanced heat transfer surfaces developed by first law analysis with criteria assessing the merits of augmentation techniques in connection with the entropy generation and exergy destruction.
2. The general evaluation criteria add new information to Bejan's EGM method assessing two objectives simultaneously. They may help to display inappropriate enhanced surfaces and assist the engineer to design better heat transfer equipment.
3. The heat transfer and fluid friction characteristics of 10 spirally corrugated tubes from three sources are used to illustrate the application of the PEC equations. The results for different design constraints show that the optimum rib-height-to-diameter ratio (e/D) for spirally corrugated tubes is about 0.04.

References

[1] R.L. Webb, E.R.G. Eckert, Application of rough surfaces to heat exchanger design, International Journal of Heat and Mass Transfer 21 (1972) 1647–1658.

[2] A.E. Bergles, A.R. Blumenkrantz, J. Taborek, Performance evaluation criteria for enhanced heat transfer surfaces, in: Fifth International Heat Transfer Conference, Tokyo, 1974, pp. 239–243.

[3] A.E. Bergles, R.L. Bunn, G.H. Junkhan, Extended performance evaluation criteria for enhanced heat transfer surfaces, Letters of Heat and Mass Transfer 1 (1974) 113–120.

[4] R.L. Webb, Performance evaluation criteria for use of enhanced heat transfer surfaces in heat exchanger design, International Journal of Heat and Mass Transfer 24 (1981) 715–726.

[5] R.L. Webb, A.E. Bergles, Performance evaluation criteria for selection of heat transfer surface geometries used in low Reynolds number heat exchangers, in: NATO Advanced Study Institute, 1981, Ankara, Turkey, Low Reynolds Number Flow Heat Exchangers, Hemisphere, Washington, DC, 1983, pp. 735–752.

[6] A. Bejan, General criterion for rating heat exchanger performance, International Journal of Heat and Mass Transfer 21 (1978) 655–658.

[7] A. Bejan, P.A. Pfister Jr., Evaluation of heat transfer augmentation techniques based on their impact on entropy generation, Letters of Heat and Mass Transfer 7 (1980) 97–106.

[8] W.R. Oulette, A. Bejan, Conservation of available work (exergy) by using promoters of swirl flow in forced convection heat transfer, Energy 5 (1980) 587–596.

[9] A. Bejan, Entropy Generation Through Heat and Fluid Flow, Wiley, New York, 1982.

[10] R.M. Nelson, A.E. Bergles, Performance evaluation criteria for tube side heat transfer enhancement of a flooded evaporator water chiller, ASHRAE Transactions 92 (1) (1986) 739–755.

[11] H. Perez-Blanco, Irreversibility in heat transfer enhancement, in: Second Low Aspects Thermal Design, 22nd National Heat Transfer Conference and Exhibition, Niagara Falls, New York, 1984, pp. 19–26.

[12] B.H. Chen, W.H. Huang, Performance evaluation criteria for enhanced heat transfer surfaces, International Communications in Heat and Mass Transfer 15 (1988) 59–72.

[13] P.K. Nad, N. Kumar, Second law optimization of convective heat transfer though a duct with constant heat flux, International Journal Energy Research 13 (1989) 537–543.

[14] R.C. Prasad, J. Shen, Performance evaluation of convective heat transfer enhancement devices using exergy analysis, International Journal of Heat and Mass Transfer 36 (17) (1993) 4193–4197.

[15] R.C. Prasad, J. Shen, Performance evaluation using exergy analysis — application to wire-coil inserts in forced convection heat transfer, International Journal of Heat and Mass Transfer 37 (15) (1994) 2297–2303.

[16] V.D. Zimparov, N.L. Vulchanov, Performance evaluation criteria for enhanced heat transfer surfaces, International Journal of Heat and Mass Transfer 37 (12) (1994) 1807–1816.

[17] W.J. Marner, A.E. Bergles, J.M. Chenoweth, On the presentation of performance data for enhanced tubes used in shell-and-tube heat exchangers, Transaction of the ASME, Journal of Heat Transfer 105 (1983) 358–365.

[18] L.W. Boyd, J.C. Hammon, J.G. Littrell, J.G. Withers, Efficiency improvement at Gallatin Unit 1 with corrugated condenser tubing, Joint Power Generation Conference, Paper No. 83-JPGC-Pwr-4, 1983.

[19] Y.I.A. Annon, Heat exchanger tubes: design data for horizontal roped tubes in steam condensers, Technical

Memorandum 3, Yorkshire Imperial Alloys, Leeds, UK, 1982.
[20] J.G. Withers, Tube-side heat transfer and pressure drop for tubes having helical internal ridging with turbulent/transitional flow of single phase fluid. Part 1: Single-helix ridging, Heat Transfer Engineering 1 (1980) 48–58.
[21] V.D. Zimparov, N.L. Vulchanov, L.B. Delov, Heat transfer and friction characteristics of spirally corrugated tubes for power plant condensers — Part 1: experimental investigation and performance evaluation, International Journal of Heat and Mass Transfer 34 (1991) 2187–2197.
[22] T.S. Ravigururajan, A.E. Bergles, An experimental verification of general correlation for single phase turbulent flow in ribbed tubes, Advances in Heat Exchanger Design, ASME HTD 66 (1986) 1–11.
[23] T.S. Ravigururajan, A.E. Bergles, Prandtl number influence on heat transfer enhancement in turbulent flow of water at low temperatures, Transactions of the ASME, Journal of Heat Transfer 117 (1995) 276–282.